

Limited Fluctuation Credibility and Variable Annuity Benefit Utilization Study Ruark Consulting, LLC

Methodology

For the Variable Annuity Benefit Utilization Study that we released in December 2009, RCL used the classical Limited Fluctuation (LF) credibility method. We believe that this method is reasonable for this purpose, as it reflects both withdrawal frequency and severity and is computationally straightforward. In this context, severity measures the amount of a withdrawal in relation to a guaranteed Benefit Base.

The LF method utilizes the Central Limit Theorem, and assumes that withdrawal frequency and severity are independent, and that withdrawals are mutually independent. RCL believes that these assumptions are reasonable for this study, and that investors' regular review of their financial plans and product performance with their financial advisor tend to mitigate the potential dependency effects of systematic withdrawal programs from year to year.

The LF method ascribes full credibility when the number of withdrawals is such that, with a high degree of probability, the actual total withdrawals should fall within a narrow confidence interval around the expected total withdrawals. The number of withdrawals is a natural choice for this purpose, as it implicitly reflects key factors that intuitively affect credibility – both credibility and the number of withdrawals tend to increase when exposure increases and when withdrawal frequency increases.

Formulas

The complete formula for N_F , the number of withdrawals needed for full credibility under the LF method, is as follows:

$$N_F = \left(\frac{x_\alpha}{k} \right)^2 \left[\frac{\sigma_f^2}{\mu_f} + \frac{\sigma_S^2}{\mu_S^2} \right]$$

where,

α = confidence level for withdrawals to be within +/-100k% of the mean experience

x_α = Standard Normal Table value such that $\Phi(x_\alpha) = \frac{1+\alpha}{2}$

σ_f^2 = variance of the frequency distribution

μ_f = mean of the frequency distribution

σ_s^2 = variance of the severity distribution

μ_s = mean of the severity distribution

The confidence level and range are subjective and not dictated by the LF method in general. RCL has applied a 95% confidence level with a +/-5% range for this study, as we believe that this is reasonable considering the likely applications of results.

In general, the LF method does not require assumptions regarding the frequency and severity distributions, and the directly calculated means and variances may be used. However, simplifying assumptions may be appropriate in certain situations.

For example, a common assumption is that withdrawal frequency follows a Poisson distribution, which implies that withdrawals occur at a constant average rate per unit of time, and that the mean equals the variance. RCL has used the Poisson assumption for this study, as we believe that it is reasonable for withdrawal frequency, and is a natural starting point for further analysis in subsequent studies.

With the Poisson assumption for withdrawal frequency as a baseline, if withdrawal severity variance is zero, then the number of withdrawals needed for full credibility reduces to $\left(\frac{x_\alpha}{k}\right)^2 = \left(\frac{1.96}{.05}\right)^2 \approx 1537$. To the extent that withdrawal severity variance is not zero, as is the case with this study, the number of withdrawals needed for full credibility is increased by the ratio $\frac{\sigma_s^2}{\mu_s^2}$ (see previous formula). Based on analysis of the study data and consideration of a range of potential distributions for both withdrawal frequency and severity, RCL believes that $\frac{\sigma_s^2}{\mu_s^2}$ will typically not exceed about 30%, so that 2000 withdrawals should be sufficient for full credibility at a 95% confidence level with a +/-5% range.

The LF method ascribes a partial credibility factor when the number of withdrawals is less than that needed for full credibility. The partial credibility factor can be derived from the assumptions for the LF method, and is equal to the square root of the ratio of experience withdrawals to the full credibility withdrawals above. For example, if there are 1125 withdrawals and full credibility corresponds to 2000 withdrawals, then the partial credibility factor is $\sqrt{1125 / 2000} = 75\%$ for these withdrawals.

References

Herzog, Thomas N., Introduction to Credibility Theory, ACTEX Publications, 1999.

Mahler, Howard C., and Dean, Curtis G., Foundations of Casualty Actuarial Science (Fourth Edition), Casualty Actuarial Society, 2001.

Credibility and the Common-Sense Actuary, Contingencies, American Academy of Actuaries, Sep/Oct 2008.